

# **Athermal Bonded Mounts**

## **Incorporating Aspect Ratio into a Closed-form Solution**

Chris Monti  
OPTI 521  
19 November, 2006

### **Abstract**

Several approaches have been used to calculate a closed-form solution for an athermal bondline for mounting optical elements. All of the previously developed closed-form solutions use the assumption that the bondline is thin with respect to the width of the bond in the axial direction. While this assumption is mathematically convenient, it is not empirically or theoretically supported. To compensate for the inaccuracies of these closed-form solutions, recent research using test data and finite element analysis has centered on generating correction factors that are applied to the closed-form solutions for a zero-stress bond. In this paper an alternative closed-form solution is presented that incorporates the bond aspect ratio. This formula is compared to the empirical results of a finite element analysis (FEA) study. An example case is used to compare the results provided by the different methods for calculating the ideal bond thickness.

### **Symbol Definitions**

|            |  |
|------------|--|
| $\alpha$   | Coefficient of thermal expansion (CTE) |
| $\nu$      | Poisson ratio of bond material         |
| $\epsilon$ | Normal strain (strain)                 |
| $\sigma$   | Normal stress (stress)                 |
| $\delta$   | Deflection from stress-free state      |
| $h$        | Bond thickness (radial direction)      |
| $L$        | Bond width (axial direction)           |
| $T$        | Temperature                            |
| $\Delta$   | Change in a variable                   |
| $r$        | Subscript: Radial direction            |
| $\theta$   | Subscript: Tangential direction        |
| $z$        | Subscript: Axial direction             |
| $o$        | Subscript: Lens (optical)              |
| $b$        | Subscript: Bond                        |
| $c$        | Subscript: Cell                        |

## 1. Introduction

Determining the ideal stress-free athermal bond has been the topic of several papers since the subject was first addressed by Bayar<sup>1</sup>. The goal is to size the bond thickness so that the expansion of the bonding material matches the difference in the growth of the lens cell and the lens. Most solutions are derived from Hooke's Law matrix for three dimensional stress. Referring only to normal stresses, Hooke's Law relates the stresses on a body in three directions to the three strains on that body.

This paper will first compare the existing derivations for the athermal bond thickness that are based on Hooke's Law. The different equations are a result of different assumptions of the way that the epoxy is constrained. Second, an assumption previously not considered is used to generate new limiting case for the athermal bond thickness. It will be shown that the limiting upper and lower bounds for athermal bond thickness are provided, respectively, by this new equation and the van Bezooijen equation. The new upper bound will be called the "Modified van Bezooijen equation". Next, a derivation will be presented for a new approximation for the ideal athermal bond thickness that spans the space between the two limiting equations. This new approximation takes into account the ratio of the bond width to the bond thickness, a factor that has previously been ignored except for empirically determined compensating correction factors. Finally, the new approximation is compared to the results of finite element analysis (FEA).

For consistency, most diagrams and language in this paper reflect the affects of a system that is *heated* from its original bonded temperature. All affects and equations also apply as the system is cooled.

## 2. Background Information

The equations for athermal bond thickness that can be derived directly from Hooke's Law are listed in table 1. The equations are listed in the order of the complexity of the assumptions used to generate them; this is also the order of increasing accuracy. A complete list of the assumptions used for the equations discussed in this paper is found in appendix A.

Table 1: Existing closed-formed solutions for athermal bond thickness

| Equation       | Major Assumption   |
|----------------|--|
| Bayar          | No axial or tangential strain on the epoxy   |
| Modified Bayar | Complete constraint of epoxy in axial and tangential directions                            |
| van Bezooijen* | Epoxy is fully constrained to the lens and the cell in the axial and tangential directions |

\*The van Bezooijen equation is sometimes referred to as the Muench equation but was originally derived by Roel van Bezooijen.

Another solution was created by DeLuzio. DeLuzio employed a different approach than simply using Hooke's Law. While DeLuzio's equation is mathematically very different than van Bezooijen's, the calculated ideal bond

thickness from the two equations is almost identical. DeLuzio's solution will only be discussed in this paper for comparison to the van Bezooijen equation and the other equations derived in this paper; it will not be derived or discussed in detail.

The principle of superposition will be used throughout this paper. For each derivation the same principle will be used: the bond material will grow in three axes and then the affects of "pushing the material," or constraining it, in one or two directions will be superimposed to calculate the total growth in the third direction. The first of the three superimposed affects is the thermal expansion of the material in all directions, see figure 1.

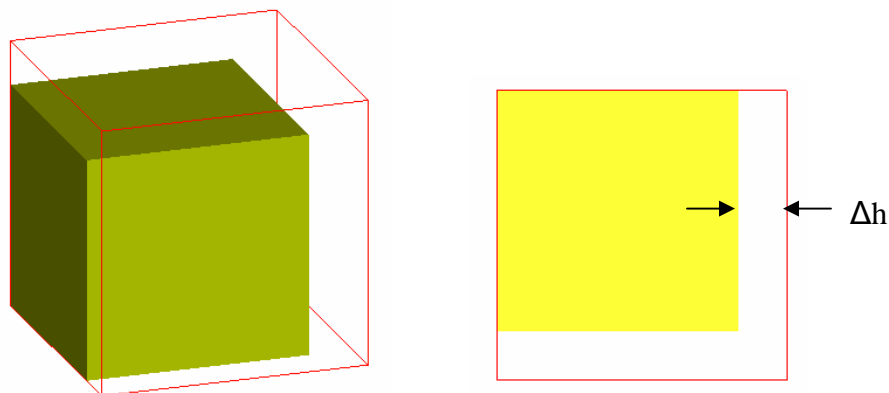


Figure 1: Thermal expansion of a unit of volume

The second affect is from compressing the cube, which is now larger, in one or two of the axes. Poisson's ratio is the material property that dictates how much the material will compress and how much it will be forced out in the directions that it is not being compressed, see figure 2.

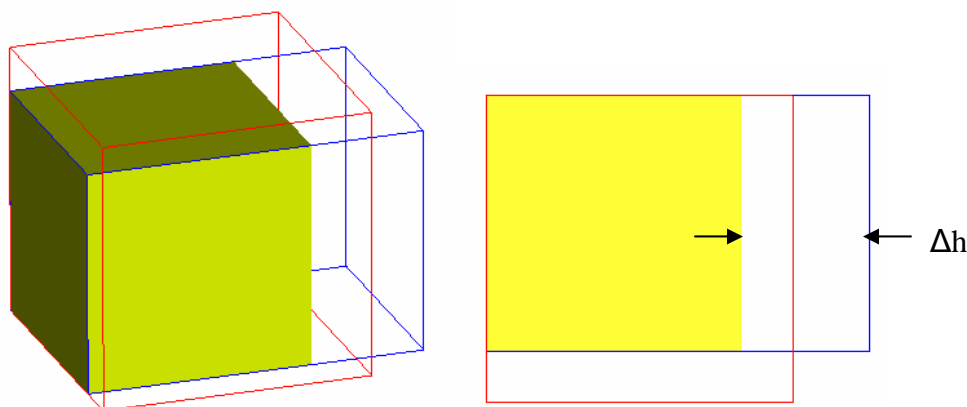


Figure 2: Constrained thermal expansion (blue); isometric and top views of a unit volume

In figure 2 the thermal expansion is shown to be fully constrained in two directions. It may also be constrained in only one direction or partially constrained in one

direction. For the limiting Poisson's ratio of .5, the problem becomes one of volume conservation; the material is incompressible.

### 3. Bayar's Equation

The simplest of the three equations commonly used for calculating bondline thickness is the Bayar equation. Bayar only looks at the radial thermal expansion and ignores the affects of constraining the bond axially and tangentially. Figure 1 defines the terms that will be used for Bayar's Equation and all other equations.

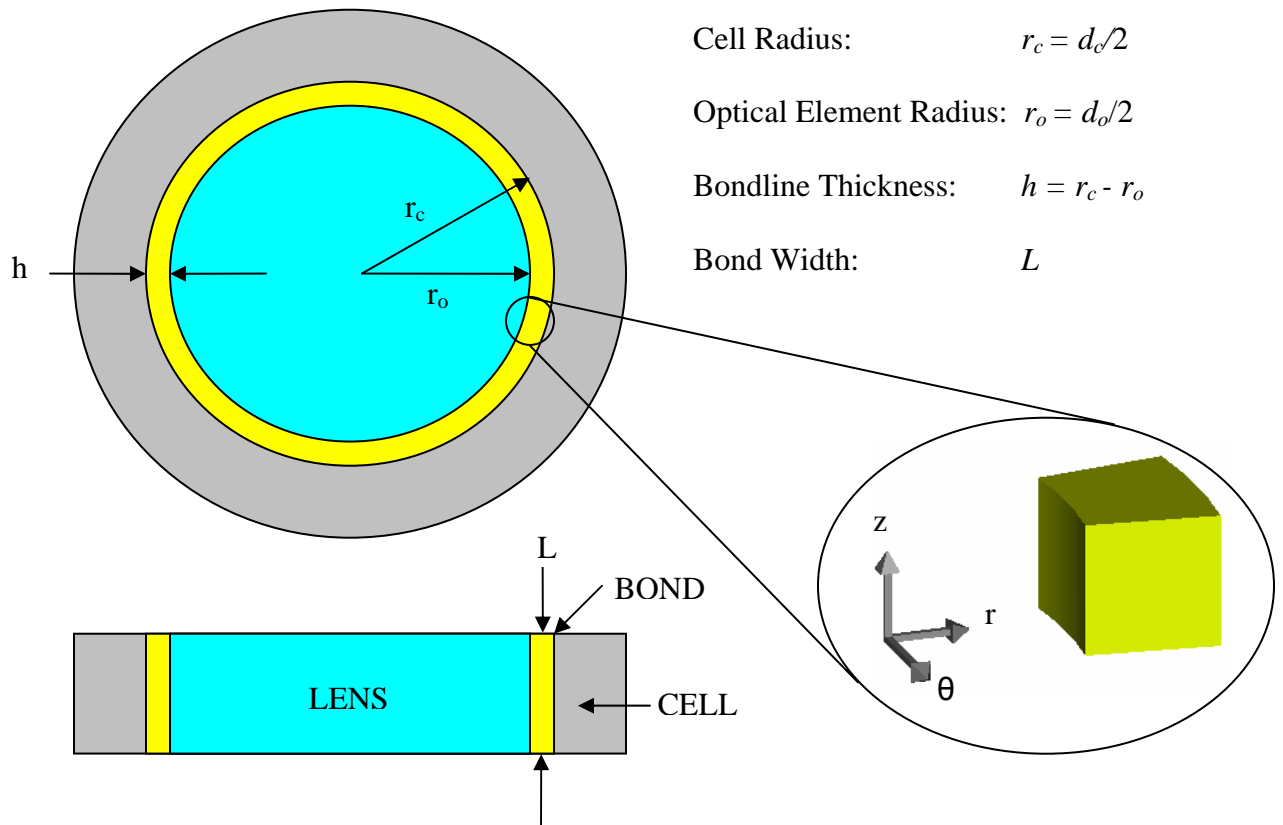


Figure 1. Typical Athermal Lens Mount

Note that in figure 1 the lens bond width,  $L$ , does not necessarily need to equal the size of the lens or the mount. It is drawn this way to simplify figures later in the document. The Bayar equation is derived by solving for the bondline thickness when equating the change in the bondline thickness over temperature to the difference in the changes of the cell and the optical radii over temperature:

$$\begin{aligned} \Delta h &= \Delta r_c - \Delta r_o \\ h\alpha_b\Delta T &= (r_o + h)\alpha_c\Delta T - r_o\alpha_o\Delta T. \end{aligned} \tag{1}$$

Solving for  $h$ , it can be seen that the thickness of the bond is a function of the optical radius and the three CTEs, as follows:

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c} \quad (2)$$

Equation (1) can also be used to calculate the radial strain, if it is not assumed to be zero, where radial strain is a function of radial deflection. The radial deflection,  $\delta h$ , is defined as the change in thickness of the bond material from its unconstrained thickness with any given change in temperature:

$$\delta h = \Delta T(h(\alpha_b - \alpha_c) - r_o(\alpha_c - \alpha_o))$$

The radial strain,  $\varepsilon_r$ , is given by the ratio of the deflection to the radial distance:

$$\varepsilon_r = \frac{\delta h}{h} = \Delta T \left( \alpha_b - \alpha_c - \frac{r_o}{h}(\alpha_c - \alpha_o) \right) \quad (3)$$

The resulting equation (3) will be used later in the paper.

Two facts are worthy of note at this time. First, if it were possible to match the CTEs of all three of the materials, then any bond thickness is allowable. Unfortunately this is typically not possible because of the inherent properties of the available materials and because of cost limitations. For most situations, and for the purposes of this paper, we assume that the CTEs of the three materials are different. Next we know that the CTE of the cell must be less than that of the optical element; if this were not the case, then the CTE of the bonding material would need to be negative. Since negative CTE bond materials are generally not available, it follows that  $\alpha_c > \alpha_o$ . Finally, the CTE of the bond must be the greatest CTE of the three because its thickness will always be less than the inner diameter of the cell. Accordingly, the following relationship must apply:

$$\alpha_b > \alpha_c > \alpha_o$$

We are fortunate that materials are readily available that meet this criteria. Metals are typically used as cell materials and glasses are commonly used for lenses and as mirror substrates. Most metals have about twice the CTE of most glasses. Bonding materials tend to have CTEs that are an order of magnitude greater than both the lens and the cell.

#### 4. Modified Bayar – Including Bulk Affects

In order to derive the several other formulas for athermal bond thickness, Hooke's Law for stress will be used. Hooke's Law is frequently presented as a

matrix that defines all six of the normal and shear stresses, but in this case we are only concerned with radial stress. Hooke's Law reduces to equation (4) for radial stress:

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)} [(1-\nu)\varepsilon_r + \nu(\varepsilon_z + \varepsilon_\theta)] \quad (4)$$

The same methodology will be used for all of the derivations that follow: expressions for the three strains will be substituted into Hooke's Law, equation (4), and the resulting equation will be solved for the zero radial stress condition. For all derivations, equation (3) is substituted for the radial strain. For consistency, Bayar's original equation is derived again in this way. The complete derivation, showing all of the algebraic steps will be shown only for the van Bezooijen equation; the others are very similar.

### Bayar

Bayar considers the strain in the axial and tangential directions to be zero. In other words, the bond layer is free to expand or contract in these directions. Substituting equation (3) into equation (4) and setting  $\varepsilon_z$  and  $\varepsilon_\theta$  to zero results in equation (5) for radial stress.

$$\sigma_r = \frac{E\Delta T}{(1+\nu)(1-2\nu)} \left[ (1-\nu) \left( \alpha_b - \alpha_c - \frac{r_o}{h} (\alpha_c - \alpha_o) \right) \right] \quad (5)$$

The radial stress is set to zero and the resulting equation is solved for the bond thickness:

$$\left( \alpha_b - \alpha_c - \frac{r_o}{h} (\alpha_c - \alpha_o) \right) = 0$$
$$h = \frac{r_o (\alpha_c - \alpha_o)}{\alpha_b - \alpha_c} \quad (6)$$

The resulting equation (6) is identical to equation (2) derived earlier.

### Modified Bayar

The Modified Bayar equation, as presented by Herbert<sup>4</sup>, affords a major improvement over the original Bayar equation. The Modified Bayar equation accounts for strains normal to the radial direction and it incorporates the Poisson ratio. The assumption used for this equation is that the strains in the tangential and axial directions are equal to the expansion of the bond layer in those directions. In other words, the bond is fully constrained to its original unheated size in both the

axial and tangential directions. Again using equation (3) in equation (4) with the additional assumption regarding the tangential and axial strains, the Modified Bayar equation and the solution  $h$  are as follows:

$$\varepsilon_z = \varepsilon_\theta = \Delta T \alpha_b$$

$$\sigma_r = \frac{E\Delta T}{(1+\nu)(1-2\nu)} \left[ (1-\nu) \left( \alpha_b - \alpha_c - \frac{r_o}{h} (\alpha_c - \alpha_o) \right) + 2\nu \alpha_b \right] \quad (7)$$

$$h = \frac{r_o (\alpha_c - \alpha_o)}{\frac{1+\nu}{1-\nu} \cdot \alpha_b - \alpha_c} \quad (8)$$

Figure 3 shows the different assumptions of the Bayar and Modified Bayar equations. The axial and tangential constraint used for the Modified Bayar equation results in greater radial bond growth than the unconstrained assumption that Bayar used for calculating equation (6). The increased growth causes the calculated ideal athermal bond to be thinner. This can be seen in equation (8), as the denominator gets bigger when compared to the Bayar equation (6).

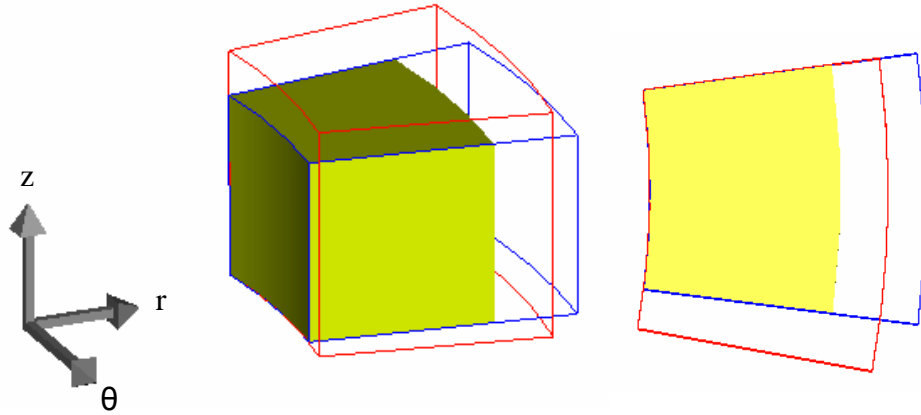


Figure 3: Comparison of Bayar (red) and Modified Bayar (blue) assumptions

## 5. Van Bezooijen Equation – Including the CTE of the Lens and Cell

Van Bezooijen<sup>7</sup> improves the approximation of the Modified Bayar equation by accounting for the expansion of the lens and the lens cell in the tangential and axial directions. The strain in these directions is described in equation (9). The strain is derived for the axial direction but also applies to the tangential direction.

$$\varepsilon_z = \frac{\delta L}{L} = \frac{1}{L} \left( L\alpha_b \Delta T - \frac{L\alpha_o \Delta T + L\alpha_c \Delta T}{2} \right)$$

$$\varepsilon_z = \varepsilon_\theta = \Delta T \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \quad (9)$$

This formula uses the average change in the size of the lens and the cell to approximate the reduction in strain from the Modified Bayar assumption. This is a good approximation considering the bond is thin compared to the radial distance from the lens axis. Figure 4 graphically shows the nature of the affects of temperature change on the system. The figure is not to scale.

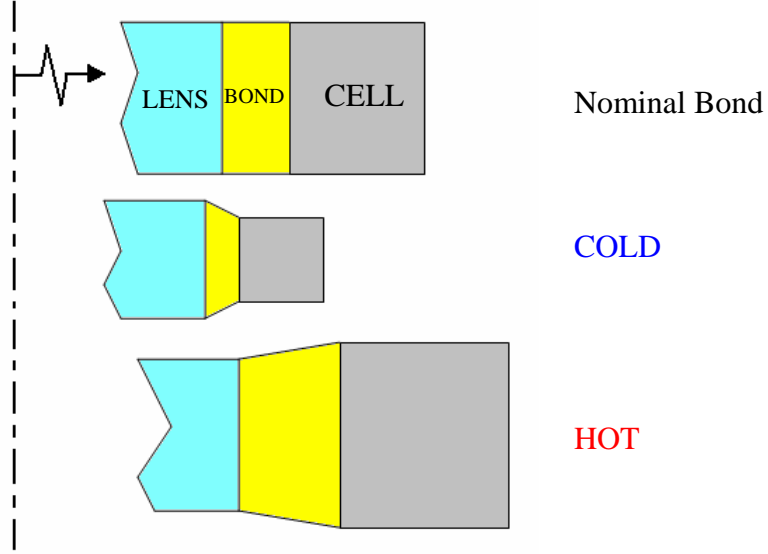


Figure 4: Representation of the deformation of the lens, cell, and bond used when calculating the van Bezooijen equation; cross section is shown, not to scale

A complete derivation of the van Bezooijen equation follows. Once again, equation (3) is substituted into (4) for the radial strain. In this case, equation (9) is substituted for the axial and tangential strains.

$$\sigma_r = \frac{E\Delta T}{(1+\nu)(1-2\nu)} \left[ (1-\nu) \left( \alpha_b - \alpha_c - \frac{r_o}{h} (\alpha_c - \alpha_o) \right) + 2\nu \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \right] \quad (10)$$

The stress is set to zero and the equation is simplified.

$$(1-\nu) \left( \alpha_b - \alpha_c - \frac{r_o}{h} (\alpha_c - \alpha_o) \right) = -2\nu \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right)$$

$$-\frac{r_o}{h} (\alpha_c - \alpha_o) = \frac{-2\nu}{(1-\nu)} \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) - \alpha_b + \alpha_c$$

$$r_o (\alpha_c - \alpha_o) = \left[ \frac{2\nu}{(1-\nu)} \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) + \alpha_b - \alpha_c \right] h$$



$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{2\nu}{1-\nu} \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right)} \quad (11)$$

Figure 5 compares the van Bezooijen assumption with the Modified Bayar assumption. It can be seen that for the van Bezooijen assumption the lens cell expands, allowing the bond material to expand slightly as well. For simplicity, a zero CTE lens is used for the comparison. Figure 5 also shows that the bond expands slightly less in the radial direction than it does under the Modified Bayar assumption. The reduced growth results in a calculated ideal bond thickness that is greater than the ideal thickness calculated by the Modified Bayar equation.

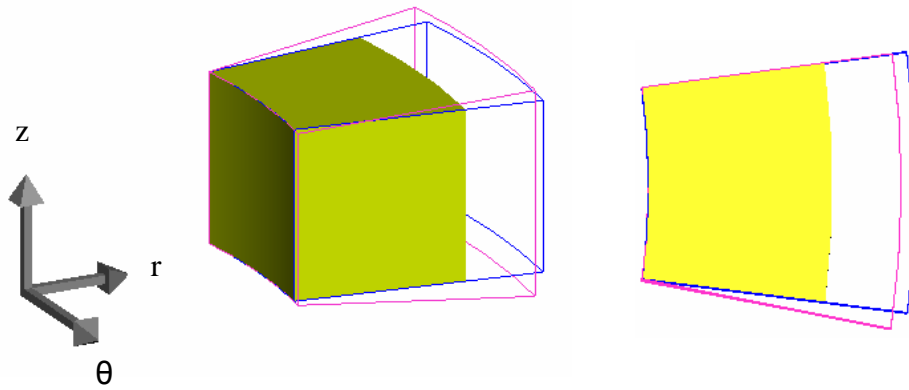


Figure 5: Comparison of Modified Bayar (blue) and van Bezooijen (pink) assumptions; isometric and top views shown

An example system is compared in figure 6. The graph compares the calculated bond thickness using the Bayar, Modified Bayar, van Bezooijen, and DeLuzio, and equations. A dramatic difference can be seen between the Bayar equation and the others. The original Bayar equation calculates a thicker bondline because it does not include the expansion of the bond radially due to constraining the axial and tangential directions. The Modified Bayar curve sits below the others because it does not incorporate the thermal expansion of the lens and cell, effectively “squeezing harder” on the bond and forcing it out further in the radial direction when the system is heated. The van Bezooijen and DeLuzio curves – virtually on top of each other – provide for the thermal expansion of the lens and cell, and thus calculate a slightly larger thermal bond. The DeLuzio equation is shown in appendix B.

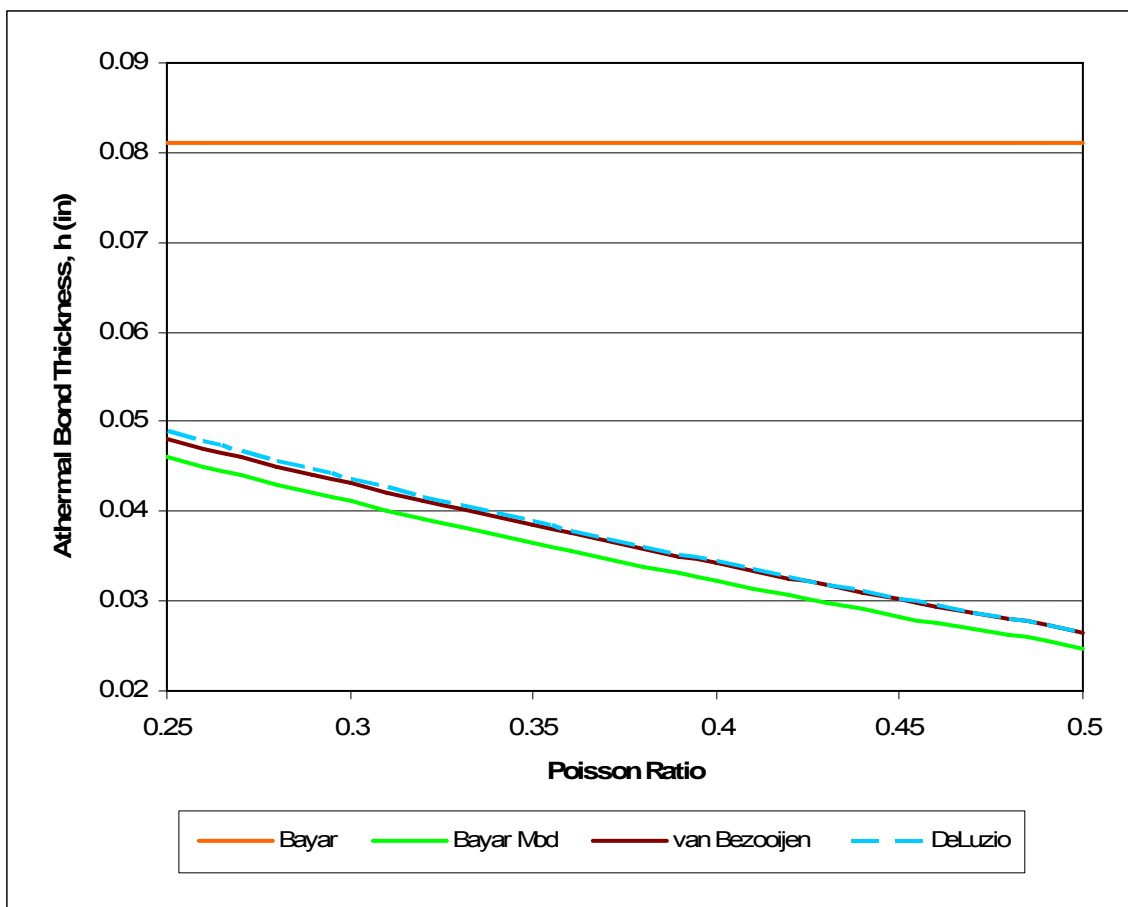


Figure 6: Plot of Bayar, Modified Bayar, van Bezooijen, and DeLuzio equations for an example system across the full span of typical bond material Poisson ratios

The van Bezooijen equation is the closest to correctly accounting for all of the constraints on the bond, making it the most accurate of the equations that can be derived from Hooke's Law. The assumption of van Bezooijen is that the bond is fully constrained to the lens and the cell. In reality this is not the case, the bond is allowed to bulge or shrink at its exposed surfaces. Because of its assumption of *complete* constraint, this equation does not calculate the ideal bond thickness, but rather it serves as a limit. Van Bezooijen slightly over constrains the bond, causing to grow more than it actually does and calculating too thin of a bond. The lower limit for the ideal bond thickness is therefore the thickness calculated by the van Bezooijen equation.

## 6. Modified van Bezooijen – The Upper Limit

The van Bezooijen was shown to be the lower limit to the ideal bond thickness; an upper limit is also needed. The upper limit is derived by making the opposite assumption of the van Bezooijen for axial strain. For this derivation the bond will be assumed to be completely unconstrained in the axial direction. This means that the axial strain is assumed to be zero. The tangential and radial strains are the same as for the original van Bezooijen derivation.

$$\varepsilon_z = 0$$

$$\sigma_r = \frac{E\Delta T}{(1+\nu)(1-2\nu)} \left[ (1-\nu) \left( \alpha_b - \alpha_c - \frac{r_o}{h} (\alpha_c - \alpha_o) \right) + \nu \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \right] \quad (12)$$

$$h = \frac{r_o (\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{\nu}{1-\nu} \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right)} \quad (13)$$

The difference between the von Bezooijen equation (11) and this newly proposed modified equation (13) is a factor of 2 on the third term in the denominator. The van Bezooijen and Modified van Bezooijen equations define the lower and upper bounds for the correct bond thickness.

The factor that determines where the actual ideal bond thickness lies between these two extremes is the bond aspect ratio. All of the standard bond thickness equations (Bayar, Modified Bayar, and van Bezooijen) neglect the impact of the aspect ratio on the performance of the bond. The aspect ratio is defined as the ratio of the bond width to the bond thickness.

$$R_{aspect} = \frac{L}{h} \quad (14)$$

The standard equations all assume a large aspect ratio for the bond so the edge affects of the exposed surfaces are negligible. However, if the aspect ratio is small, the axial strain can no longer be assumed to be completely constrained in this way. For extremely low aspect ratios the strain in axial direction can be set to zero because as the width of the bond,  $L$ , approaches zero, bulge of the bond at temperature is the dominant affect. For low aspect ratios the axial constraint of being adhered to the lens and cell can be neglected; the modified van Bezooijen represents this case.

## 7. Aspect Ratio Approximation

Now that the two limiting cases have been defined, a relationship is needed that spans the region between them. The aspect ratio is required in order to derive a new equation that spans this space. To incorporate the aspect ratio, we let the Modified van Bezooijen equation represent an aspect ratio of one. In other words, the assumption is made that, if the width of the bond is equal to the thickness of the bond, then the bond behaves as though it is not constrained in the axial direction. The “fraction of axially constrained bond” is then defined as follows:

$$R_{constrained} = \frac{L-h}{L} = 1 - \frac{h}{L} = 1 - R_{aspect}^{-1} \quad (15)$$

This relationship is shown graphically in figure 7, where the unconstrained portions of the bond are highlighted. This is clearly an approximation; in reality, the edges of the bond are not completely unconstrained and the center of the bond is not fully constrained.

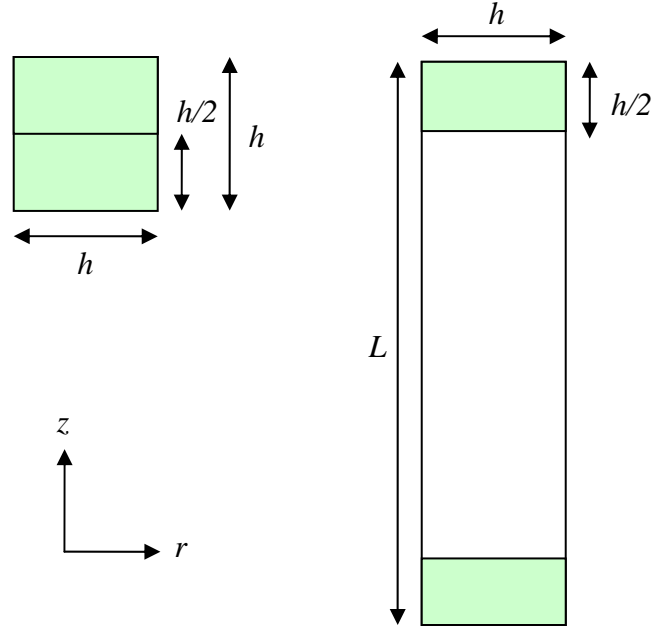


Figure 7: Bond with an aspect ratio of 1 compared to a more typical bond cross section

The formula for stress then becomes:

$$\sigma_r = \frac{E\Delta T}{(1+\nu)(1-2\nu)} \left[ (1-\nu) \left( \alpha_b - \alpha_c - \frac{r_o}{h} (\alpha_c - \alpha_o) \right) + \nu \left( 2 - \frac{h}{L} \right) \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \right] \quad (16)$$

$$h = \frac{r_o (\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{\nu}{1-\nu} \left( 2 - \frac{h}{L} \right) \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right)} \quad (17)$$

Note the coefficient of the last term is  $\left( 2 - \frac{h}{L} \right)$ , which can assume values between 1

and 2 for values of  $L$  between  $h$  and infinity. Recall that the coefficient of this last term in the denominator of the van Bezooijen equation, (11), is 2 and coefficient of the same term the Modified van Bezooijen equation, (13), is 1. Computations of  $h$  will indeed be bounded by those two equations. Equation (17) is shown to demonstrate the similarities between this formula and the previously derived formulas for bond thickness, it does not explicitly solve for  $h$ , however. The quadratic formula is required to formulate the complete closed-form solution.

$$0 = h^2 \left( \frac{-\nu}{L} \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \right) + h \left( (1-\nu)(\alpha_b - \alpha_c) + 2\nu \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \right) - r_o(1-\nu)(\alpha_c - \alpha_o)$$

$$\left. \begin{aligned} a &= \frac{-\nu}{L} \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \\ b &= (1-\nu)(\alpha_b - \alpha_c) + 2\nu \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \\ c &= r_o(1-\nu)(\alpha_c - \alpha_o) \end{aligned} \right\} h = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (18)$$

The alternate solution to the quadratic formula (the solution having the minus sign before the square root) is associated with unrealistic values of the bond width and is a result of the aspect ratio approximation only being valid for bond widths between 1 and infinity. Figure 8 compares the assumptions of the van Bezooijen and aspect ratio approximation equations. As before, a zero CTE lens is used for the comparison. The aspect ratio approximation expands less in the radial direction because it is allowed to bulge at the exposed top and bottom surfaces. Accounting for this additional bulging results in a calculated ideal bond thickness that is greater than the thickness calculated by the van Bezooijen equation.

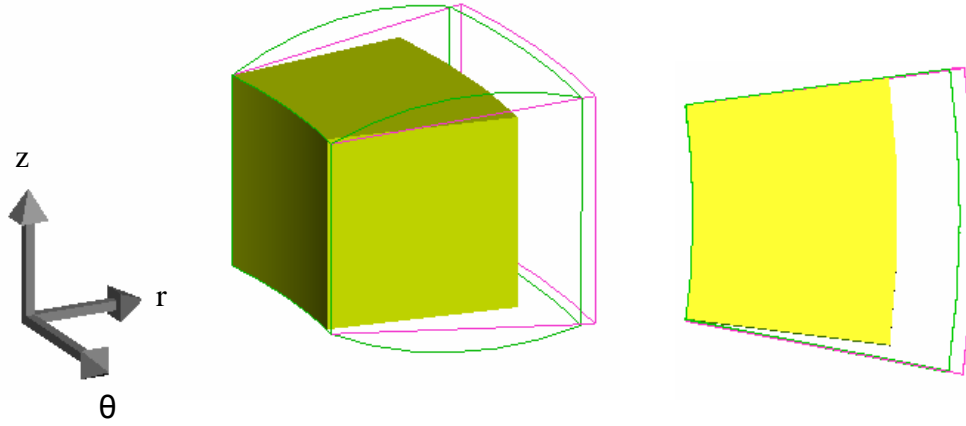


Figure 8: Comparison of van Bezooijen (pink) and aspect ratio approximation assumptions; isometric and top views shown

A cross section is shown in figure 9 that includes the lens and cell. Figure 9 demonstrates the bulging that occurs at the free surfaces when the system is heated and the shrinkage of the bond that occurs when the system is cooled.

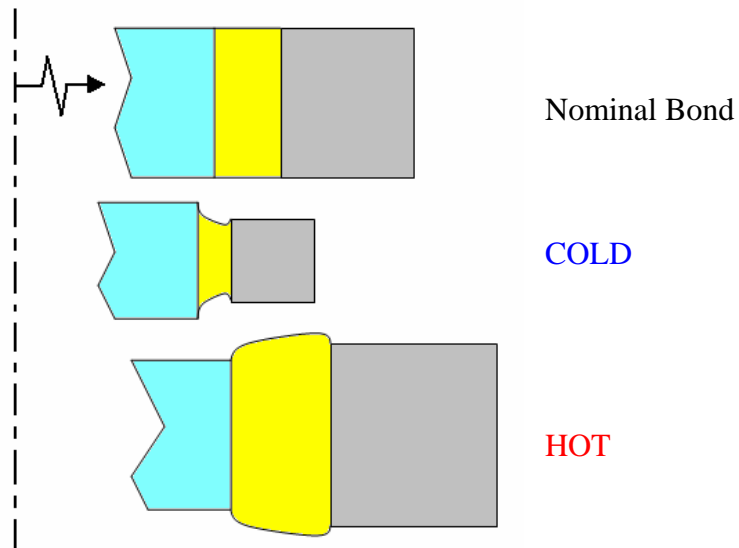


Figure 9: Representation of the deformation of the lens, cell, and bond used when calculating the aspect ratio approximation; cross section is shown, not to scale.

A graphical comparison of the van Bezooijen, Modified van Bezooijen, and aspect ratio approximation methods is shown for various aspect ratios in an example system in figure 10. As the aspect ratio approaches one, the approximate solution approaches the Modified van Bezooijen equation. The example system with the bond width equal to 0.25 is identical to the example system presented by Herbert<sup>4</sup>.

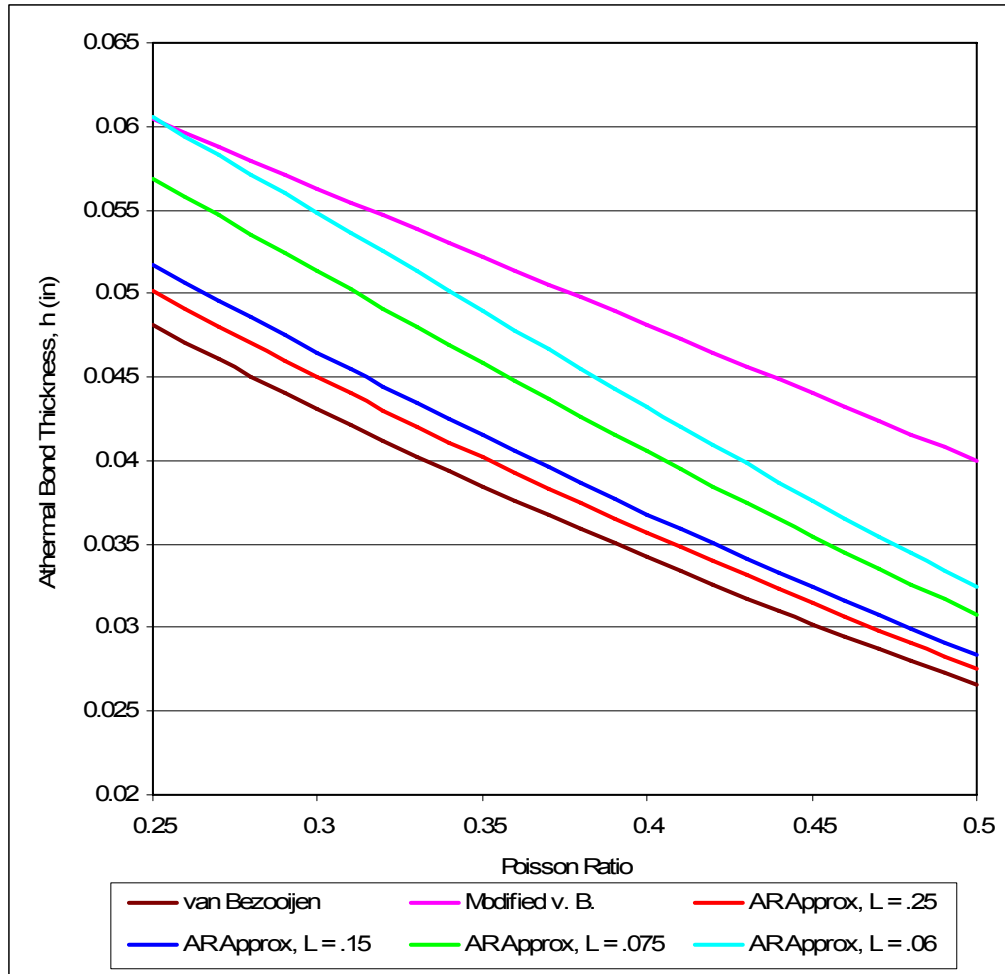


Figure 10: Comparison of van Bezooijen, modified van Bezooijen, and the aspect ratio approximation for three different aspect ratios across the full span of typical bond material Poisson ratios

## 8. Comparison to FEM Data

A modified Hooke's Law matrix was developed by Michaels and Doyle<sup>5</sup> that incorporates correction factors. Michaels and Doyle present the results of data they compiled from FEM analysis. They used the results of their study to empirically determine correction factors to be applied to Hooke's Law. The result of their work for radial stress is:

$$\sigma_r = \frac{E}{(1+\nu)(1-2\nu)} [k_{11}(1-\nu)\varepsilon_r + k_{11}\nu(k_{12}\varepsilon_z + k_{13}\varepsilon_\theta)]$$

The correction factors are  $k_{11}$ ,  $k_{12}$ , and  $k_{13}$ , which are available in tabular form as functions of the bond aspect ratio and Poisson's ratio. Equations (3) and (9) are plugged into the radial stress equation resulting in a formula for stress in terms of the correction factors. This equation is then solved for the bond thickness.

$$\sigma_r = \frac{E\Delta T}{(1+\nu)(1-2\nu)} \left[ k_{11}(1-\nu) \left( \alpha_b - \alpha_c - \frac{r_o}{h} (\alpha_c - \alpha_o) \right) + k_{11}\nu(k_{12} + k_{13}) \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right) \right]$$

$$h = \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_c + \frac{\nu}{1-\nu}(k_{12} + k_{13}) \left( \alpha_b - \frac{\alpha_o + \alpha_c}{2} \right)} \quad (19)$$

The correction factor  $k_{11}$  drops out of the equation for bond thickness. The result is a direct connection with equation (17). The two expressions are related:

$$\frac{k_{12} + k_{13}}{2 - \frac{h}{L}}$$

The two expressions above are compared in Figure 11. The FEA data were collected for several values of the Poisson ratio, all values in the relatively narrow range of 0.45 and above.

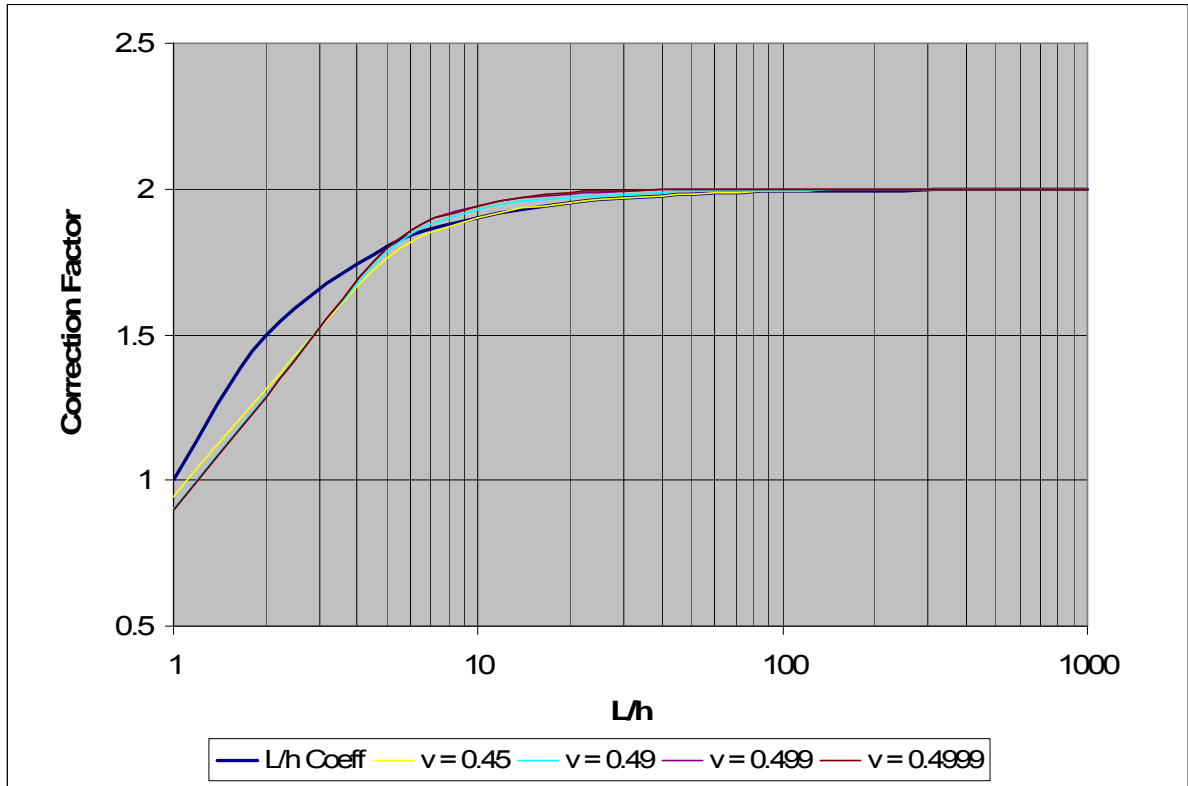


Figure 11: Comparison of the correction factor as calculated by the aspect ratio approximation to the correction factor from FEA data for several Poisson ratios



The coefficient calculated by the aspect ratio approximation is shown to be very close to the tabulated coefficient gathered from FEA data. The relationship is particularly close for aspect ratios greater than four. The benefit of the aspect ratio approximation is that it provides a closed-form solution. During the design process any values for the bond width ( $L$ ), lens diameter ( $r_o$ ), bond material Poisson Ratio ( $\nu$ ), and CTEs ( $\alpha_o$ ,  $\alpha_b$ ,  $\alpha_c$ ) can be used to calculate an approximate ideal bond thickness. There is no need to refer to a table of correction factors and to iterate to a solution.

## 9. Conclusion

A new closed-form solution for the ideal athermal bond thickness was developed. The solution was generated by incorporating the bond aspect ratio in order to span the space between the two limiting cases for athermal bond thickness. The lower limit was shown to be the equation originally developed by van Bezooijen; a newly derived upper limit was formulated by allowing for the bond to be unconstrained in the axial direction. Finally, the aspect ratio approximation solution was derived and compared to empirical data derived from FEA. The closed-form solution is shown to be quite close to the tabulated results from the FEA data.

The closed-form solution derived in this paper is the closest approximation to the ideal athermal bond thickness presented to date. It should be used for calculations when the zero radial stress condition is critical. The new equation still makes several assumptions; the assumptions of all the equations discussed in this paper are summarized in appendix A. One of the most important assumptions, common to all equations, is that there is a linear CTE for all materials. Since this assumption generally is valid only when temperature changes are relatively small, the approximation becomes less accurate when applied across wide temperature ranges. The detailed description of the assumptions used for each derivation should aid the reader in deriving approximations for more complicated geometries.

### References

1. Bayar, Mete. "Lens Barrel Optomechanical Design Principles", Optical Engineering. Vol. 20 No. 2 (April 1981)
2. Doyle, Keith, Gregory Michels, and Victor Grenberg. "Athermal Design of Nearly Incompressible Bonds", SPIE 4771, 296 (2002)
3. Gastreich, R.P. "Optical Lens Element Distortion Analysis", Raytheon Missile Systems Technical Memo. (Dec 1998)
4. Herbert, James. "Techniques for Deriving Optimal Bondlines for Athermal Bonded Mounts", SPIE 6288, 62880J (2006)
5. Michels, Gregory, and Keith Doyle. "Finite Element Modeling of Nearly Incompressible Bonds", SPIE 4771, 287 (2002)
6. Miller, Kirk A. "Non-Athermal Potting of Optics", SPIE 3786, 506 (1999)
7. Van Bezooijen, Roel. "Soft Retained AST Optics" Lockheed Martin Technical Memo
8. Vukobratovich, Daniel, et al. "Bonded Mounts for Small Cryogenic Optics", SPIE 4131, 228 (2000)

### Appendix A: Table of Athermal Bond Equations

| Name of Equation                         | #    | Assumptions   | Benefit   |
|--|------|---|---|
| Bayar                                    | 2, 6 | -Unconstrained in axial and tangential directions   | First order approximation   |
| Modified Bayar                           | 8    | -Perfectly constrained in axial and tangential directions – no thermal expansion of cell and lens in those directions<br>-No axial bulging of bond<br>-Low aspect ratio | Much closer to correct solution than Bayar  |
| Van Bezooijen (Muench)                   | 11   | -Perfectly constrained to the expanding and contracting lens and cell<br>-No axial bulging of bond<br>-Low aspect ratio   | Closer to the real solution than the modified Bayar. Serves as the lower limit to the correct solution. |
| Modified van Bezooijen (Modified Muench) | 13   | -Perfectly constrained in tangential direction to the expanding and contracting lens and cell<br>-Unconstrained in the axial direction<br>-Large aspect ratio           | Serves as the upper limit to the correct solution.  |
| Aspect Ratio Approximation               | 18   | -The correct model for the aspect ratio is used   | The closest closed-form solution for athermal bondline thickness  |
| FEM Data Corrected                       | 19   | -FEM methodology is correct   | Perhaps the most accurate calculation for optimal solution  |
| All                                      | NA   | -Constant CTEs<br>-Cell and lens are infinitely stiff<br>-Zero stress bond at cure temperature (zero shrinkage)<br>-Constant Poisson ratio<br>-No thermal gradient      | NA  |

### Appendix B: The DeLuzio Equation

$$h = \left( \frac{1-\nu}{1+\nu} \right) \frac{r_o(\alpha_c - \alpha_o)}{\alpha_b - \alpha_o + \frac{(7-6\nu) \cdot (\alpha_c - \alpha_o)}{4(1-\nu)}}$$